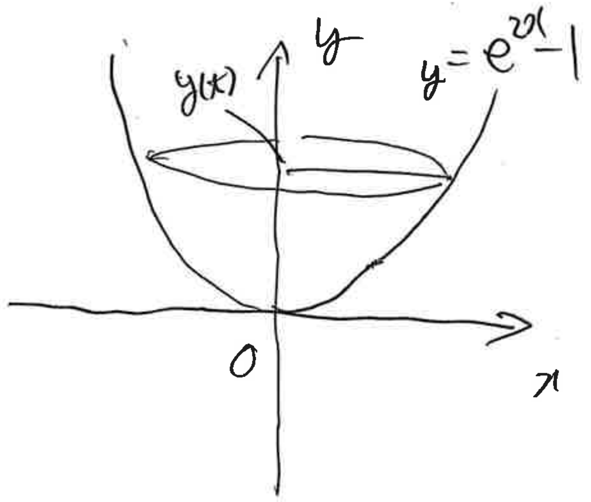
2026 鳥取大 医[1]



(1) t分後の水の体積を$V(t)$とすると

$y=e^{2x}-1$ より $x=\frac{1}{2}\log(y+1)$ より

$$V(t)=\int_0^{y(t)}\pi\left\{\frac{1}{2}\log(y+1)\right\}^2dy$$

$$\frac{d}{dt}V(t)=\frac{dy(t)}{dt}\times\frac{d}{dy(t)}\int_0^{y(t)}\frac{\pi}{4}\{\log(y+1)\}^2dy$$

$$a\pi=\frac{dy(t)}{dt}\times\frac{\pi}{4}\{\log(y(t)+1)\}^2$$

$$\therefore\ \frac{dy(t)}{dt}=\frac{4a}{\{\log(y(t)+1)\}^2}$$

求める水面の上昇する速さは $y(t)=b$ のとき

$$\frac{dy(t)}{dt}=\frac{4a}{\{\log(b+1)\}^2}\ \cdots(答)$$

(2) $S(t)=\pi\left\{\frac{1}{2}\log(y(t)+1)\right\}^2$ から

$$\frac{dS(t)}{dt}=\frac{dy(t)}{dt}\times\frac{d}{dy(t)}\frac{\pi}{4}\{\log(y(t)+1)\}^2$$

$$=\frac{4a}{\{\log(y(t)+1)\}^2}\times\frac{\pi}{4}\times2\log(y(t)+1)\times\frac{1}{y(t)+1}$$

$$=\frac{2\pi a}{(y(t)+1)\log(y(t)+1)}$$

求める水面の面積が増加する速さは $y(t)=b$ のとき

$$\frac{dS(t)}{dt}=\frac{2\pi a}{(b+1)\log(b+1)}\ \cdots(答)$$

(1) $\begin{cases} x_n = x_{n-1} + \dfrac{1}{2^{n-1}} \\ y_n = y_{n-1} + \dfrac{2}{2^{n-1}} \end{cases}$ …①

①から $x_{n+1} - x_n = \dfrac{1}{2^n}$

したがって数列$\{x_n\}$の階差数列の一般項は$\dfrac{1}{2^n}$なので，$n \geqq 2$のとき

$$x_n = x_1 + \sum_{k=1}^{n-1} \frac{1}{2^k} = 1 + \frac{1}{2} \cdot \frac{1-\left(\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}} = 1 + 2\left\{\frac{1}{2} - \left(\frac{1}{2}\right)^n\right\} = 2 - \left(\frac{1}{2}\right)^{n-1}$$

この式は$n=1$のときも成り立つ。

よって $x_n = 2 - \left(\dfrac{1}{2}\right)^{n-1}$

①から $y_{n+1} - y_n = \dfrac{2}{2^n} = \dfrac{1}{2^{n-1}}$

同様に $n \geqq 2$のとき $y_n = y_1 + \displaystyle\sum_{k=1}^{n-1} \frac{1}{2^{k-1}} = 1 + 1 \cdot \frac{1-\left(\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}} = 3 - \left(\frac{1}{2}\right)^{n-2}$

この式は$n=1$のときも成り立つ．よって$y_n = 3 - \left(\dfrac{1}{2}\right)^{n-2}$　ゆえに $P_n\left(2-\left(\frac{1}{2}\right)^{n-1}, 3-\left(\frac{1}{2}\right)^{n-2}\right)$…(答)

(2) (1)から $2 - \left(\dfrac{1}{2}\right)^{n-1} > \dfrac{3}{2}$

$$\frac{1}{2} > \left(\frac{1}{2}\right)^{n-1}$$

$\therefore$ $1 < n-1$ で $2 < n$　よって $n=3$ …(答)

(3) $\overrightarrow{P_1} \cdot \overrightarrow{P_n} = 1 \cdot \left\{2 - \left(\frac{1}{2}\right)^{n-1}\right\} + 1 \cdot \left\{3 - \left(\frac{1}{2}\right)^{n-2}\right\}$

$= 5 - \dfrac{3}{2}\left(\dfrac{1}{2}\right)^{n-2}$ …(答)

$\therefore \displaystyle\lim_{n\to\infty} \overrightarrow{P_1} \cdot \overrightarrow{P_n} = \lim_{n\to\infty}\left\{5 - \frac{3}{2}\left(\frac{1}{2}\right)^{n-2}\right\} = 5$ …(答)

(4) 点P_nと直線 $y=x$ つまり $x-y=0$ との距離 d_nは

$$d_n = \frac{\left|2-\left(\frac{1}{2}\right)^{n-1} - \left\{3-\left(\frac{1}{2}\right)^{n-2}\right\}\right|}{\sqrt{2}} = \frac{1-\left(\frac{1}{2}\right)^{n-1}}{\sqrt{2}}$$

$\therefore \displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{2}}\left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ …(答)

2026 鳥取大 医[III] 生命・検査・工・獣医[IV]

(1) $A = z\bar{z} + (1-\alpha z)(1-\overline{\alpha z})$

$= (1+|\alpha|^2)z\bar{z} - \alpha z - \bar{\alpha}\bar{z} + 1$

$= (1+|\alpha|^2)\left(z\bar{z} - \dfrac{\alpha}{1+|\alpha|^2}z - \dfrac{\bar{\alpha}}{1+|\alpha|^2}\bar{z}\right) + 1$

$= (1+|\alpha|^2)\left(z - \dfrac{\bar{\alpha}}{1+|\alpha|^2}\right)\left(\bar{z} - \dfrac{\alpha}{1+|\alpha|^2}\right) - \dfrac{|\alpha|^2}{1+|\alpha|^2} + 1$

$= (1+|\alpha|^2)\left|z - \dfrac{\bar{\alpha}}{1+|\alpha|^2}\right|^2 + \dfrac{1}{1+|\alpha|^2}$

$(1+|\alpha|^2)\left|z - \dfrac{\bar{\alpha}}{1+|\alpha|^2}\right| \geqq 0$ より

$z = \dfrac{\bar{\alpha}}{1+|\alpha|^2}$ のとき 最小値 $\dfrac{1}{1+|\alpha|^2}$ …(答)

(2) $B = z\bar{z} + (1-\alpha z)(1-\overline{\alpha z}) + (1-\beta z)(1-\overline{\beta z})$

$= (1+|\alpha|^2+|\beta|^2)z\bar{z} - (\alpha+\beta)z - (\bar{\alpha}+\bar{\beta})\bar{z} + 2$

ここで $1+|\alpha|^2+|\beta|^2 = M$ (Mは実数) とおく

$B = M\left(z\bar{z} - \dfrac{\alpha+\beta}{M}z - \dfrac{\bar{\alpha}+\bar{\beta}}{M}\bar{z}\right) + 2$

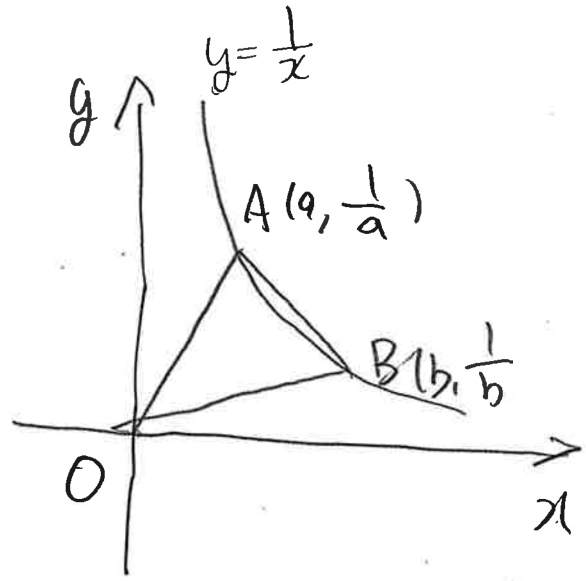
$= M\left(z - \dfrac{\bar{\alpha}+\bar{\beta}}{M}\right)\left(\bar{z} - \dfrac{\alpha+\beta}{M}\right) - \dfrac{|\alpha+\beta|^2}{M} + 2$

$= M\left|z - \dfrac{\overline{\alpha+\beta}}{M}\right|^2 + 2 - \dfrac{|\alpha+\beta|^2}{M}$

$M\left|z - \dfrac{\overline{\alpha+\beta}}{M}\right|^2 \geqq 0$ より

$z = \dfrac{\bar{\alpha}+\bar{\beta}}{1+|\alpha|^2+|\beta|^2}$ のとき最小値 $2 - \dfrac{|\alpha+\beta|^2}{1+|\alpha|^2+|\beta|^2}$ …(答)

2026 鳥取大 医 [IV]



(1) $\triangle AOB=\frac{1}{2}\left|a\times\frac{1}{b}-b\times\frac{1}{a}\right|$

$=\frac{1}{2}\left|\frac{a}{b}-\frac{b}{a}\right|$ …(答)

(2) $\frac{\pi}{4}<\theta<\frac{5}{4}\pi$ より $\sin\theta>\cos\theta$ から $0<a<b$

$\therefore\ 0<\frac{a}{b}<1<\frac{b}{a}$ から $f(\theta)=\frac{1}{2}\left(\frac{b}{a}-\frac{a}{b}\right)$

ここで $\frac{b}{a}=t$ とおくと $t>1$ で

$g(t)=\frac{1}{2}\left(t-\frac{1}{t}\right)$ とおくと $g'(t)=\frac{1}{2}\left(1+\frac{1}{t^2}\right)>0$

$g(t)$ は $t>1$ で 単調増加。…①

また $t=\frac{b}{a}=\frac{\sin\theta+\sqrt{2}}{\cos\theta+\sqrt{2}}=h(\theta)$，とおくと

$h'(\theta)=\frac{\cos\theta(\cos\theta+\sqrt{2})-(\sin\theta+\sqrt{2})\cdot(-\sin\theta)}{(\cos\theta+\sqrt{2})^2}=\frac{1+2\sin(\theta+\frac{\pi}{4})}{(\cos\theta+\sqrt{2})^2}$

$h'(\theta)=0$ となると $\sin(\theta+\frac{\pi}{4})=\frac{-1}{2}$

$\frac{\pi}{2}<\theta+\frac{\pi}{4}<\frac{3}{2}\pi$ から $\theta+\frac{\pi}{4}=\frac{7}{6}\pi$ $\therefore\ \theta=\frac{11}{12}\pi$

θ	$\frac{\pi}{4}$	…	$\frac{11}{12}\pi$	…	$\frac{5}{4}\pi$
$h'(\theta)$		$+$	0	$-$	
$h(\theta)$	1	↗	最大	↘	1

$h(\frac{11}{12}\pi)=\frac{\sin\frac{11}{12}\pi+\sqrt{2}}{\cos\frac{11}{12}\pi+\sqrt{2}}=\frac{\frac{\sqrt{6}-\sqrt{2}}{4}+\sqrt{2}}{\frac{-\sqrt{6}-\sqrt{2}}{4}+2}=\frac{3\sqrt{2}+\sqrt{6}}{3\sqrt{2}-\sqrt{6}}=2+\sqrt{3}$

よって $1<t\leqq 2+\sqrt{3}$

$g(2+\sqrt{3})=\frac{1}{2}\left(2+\sqrt{3}-\frac{1}{2+\sqrt{3}}\right)=\sqrt{3}$

これと①より

$\theta=\frac{11}{12}\pi$ のとき 最大値 $\sqrt{3}$ …(答)